Problem 3:

Given, =; C(s) = ;

where =1, T=0.4, α=0.5, β=1.216, =194.4, =0.1817

= = () = () ();

= C(s)

= ( ) () ()

= ()() ( ()

Putting the given values

= (0.187) ()()

Now separating all the basic terms from = () () ( (), the asymptotic calculation of , ( is shown below

1. For , |)| = 20log ()

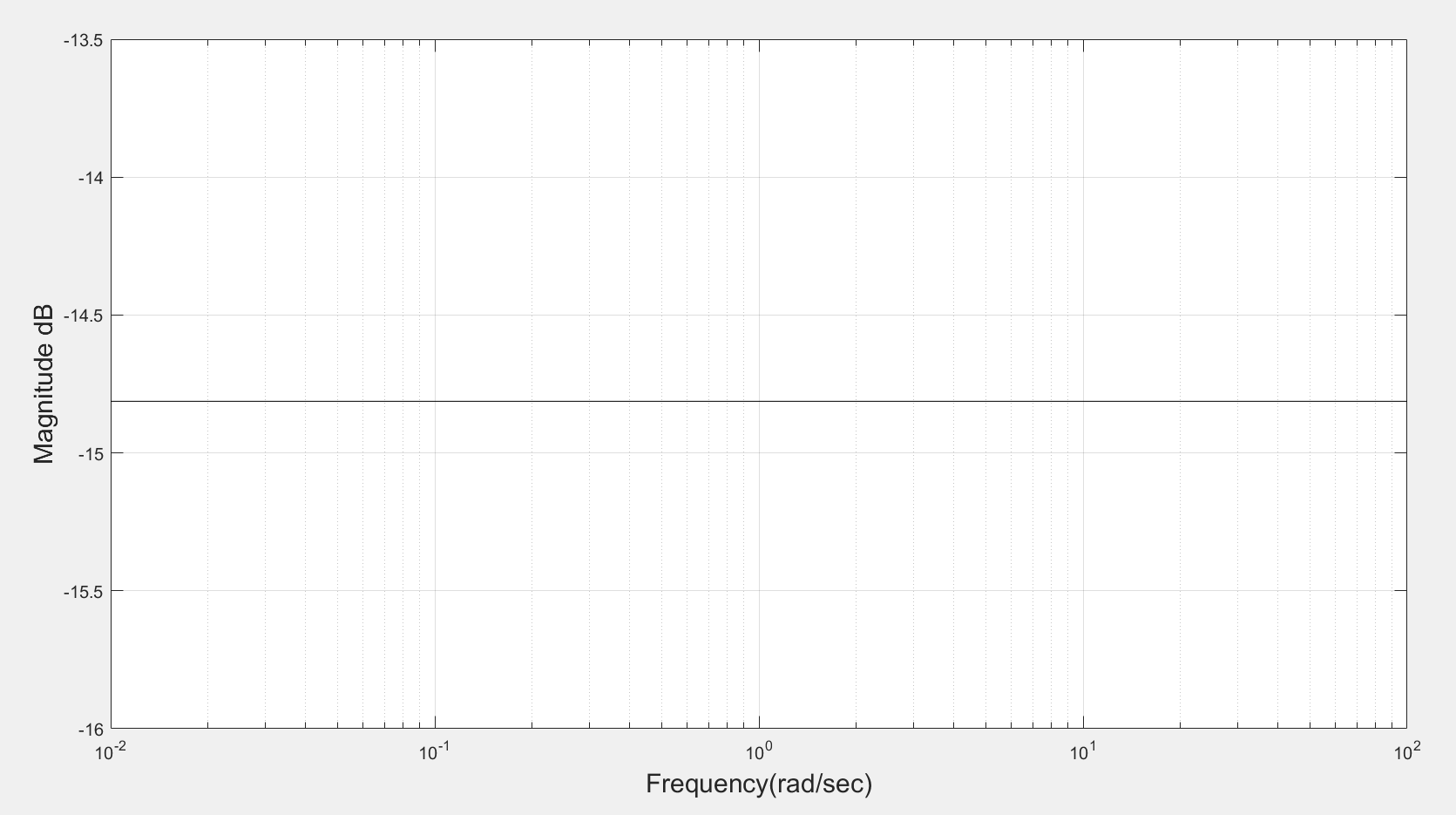


Figure 1: Asymptotic magnitude bode plot for constant gain,

1. For, (

Fractional zero transfer function is given by )=…... (1)

Put s = jω, in equation (1) results into ) =…… (2)

Magnitude in dB is given by |) | dB = 20log ( )

**Calculation procedure**

==

Applying De Moivre’s theorem in above equation we get

= …… (3)

Put equation (3) in (2) we get

) =+

= +

Magnitude, |) | =

=

Now, Magnitude in dB, |) | dB = 20log

In the sum ), dominates at lower frequencies whereas dominates at higher frequencies.

For approximation we consider = .We obtain corner frequency, =

Now, following approximation of magnitude is obtained:

1. For ω ≤ , |) | dB = 20log = 20log||
2. For ω >, |) | dB = 20log= 20βlog ω

**Procedure**

* Compute the corner frequency = and locate the point at magnitude 20log||.
* Draw a slope 0 dB/decade for ω ≤ and a line with slope 20β dB/decade for ω>

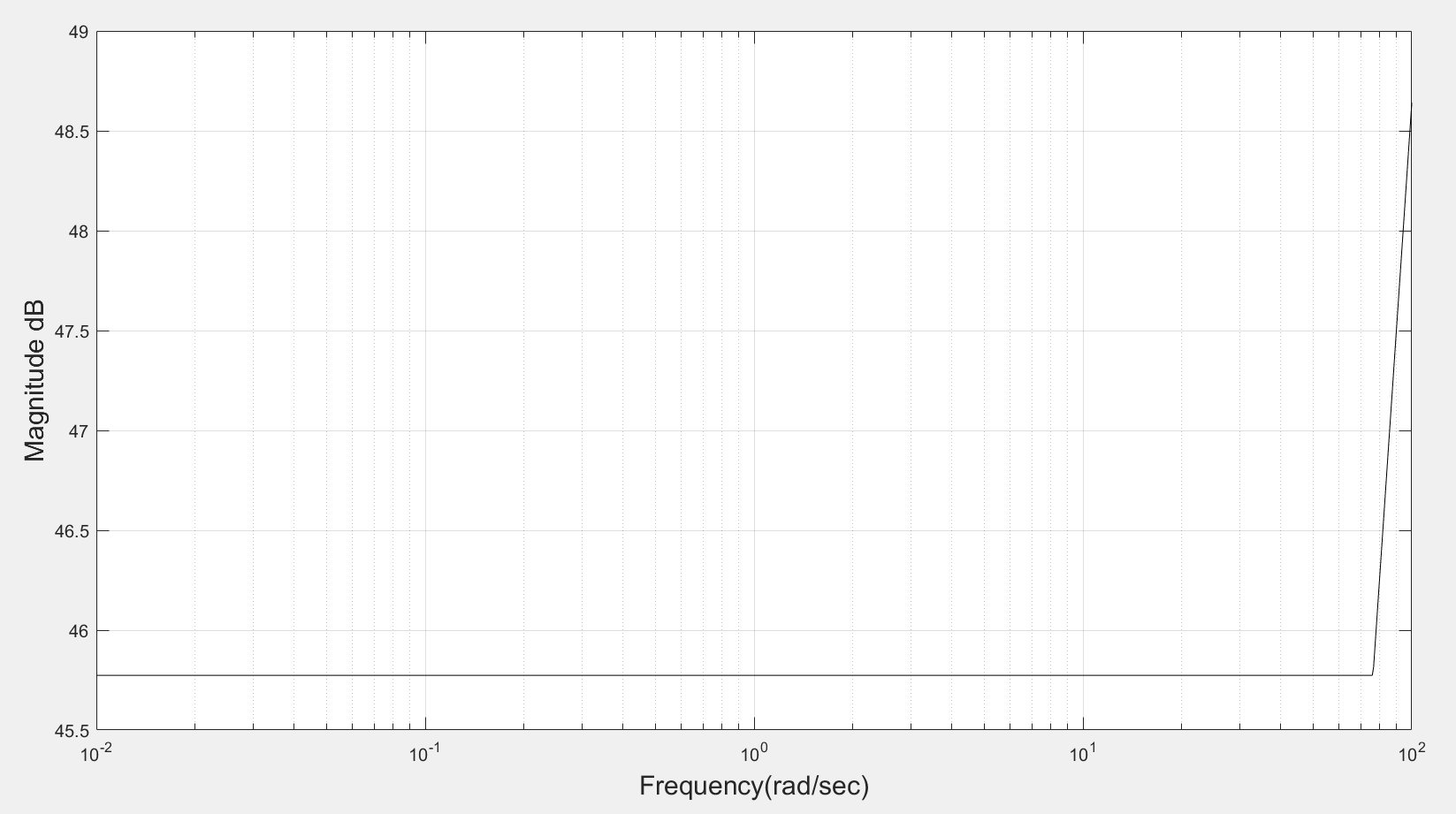


Figure 2: Asymptotic magnitude bode plot for (

1. For,

Fractional zero transfer function is given by ) =…… (1)

Put s = jω, in equation (1) results into ) = ……. (2)

Magnitude in dB is given by |)| dB= -20βlogω

**Calculation procedure**

==

Applying De Moivre’s theorem in above equation we get

= . …… (3)

Put equation (3) in (2) we get

) =

Magnitude, |) | = =

Magnitude in dB is given by |T(jω) | dB= -20βlogω.

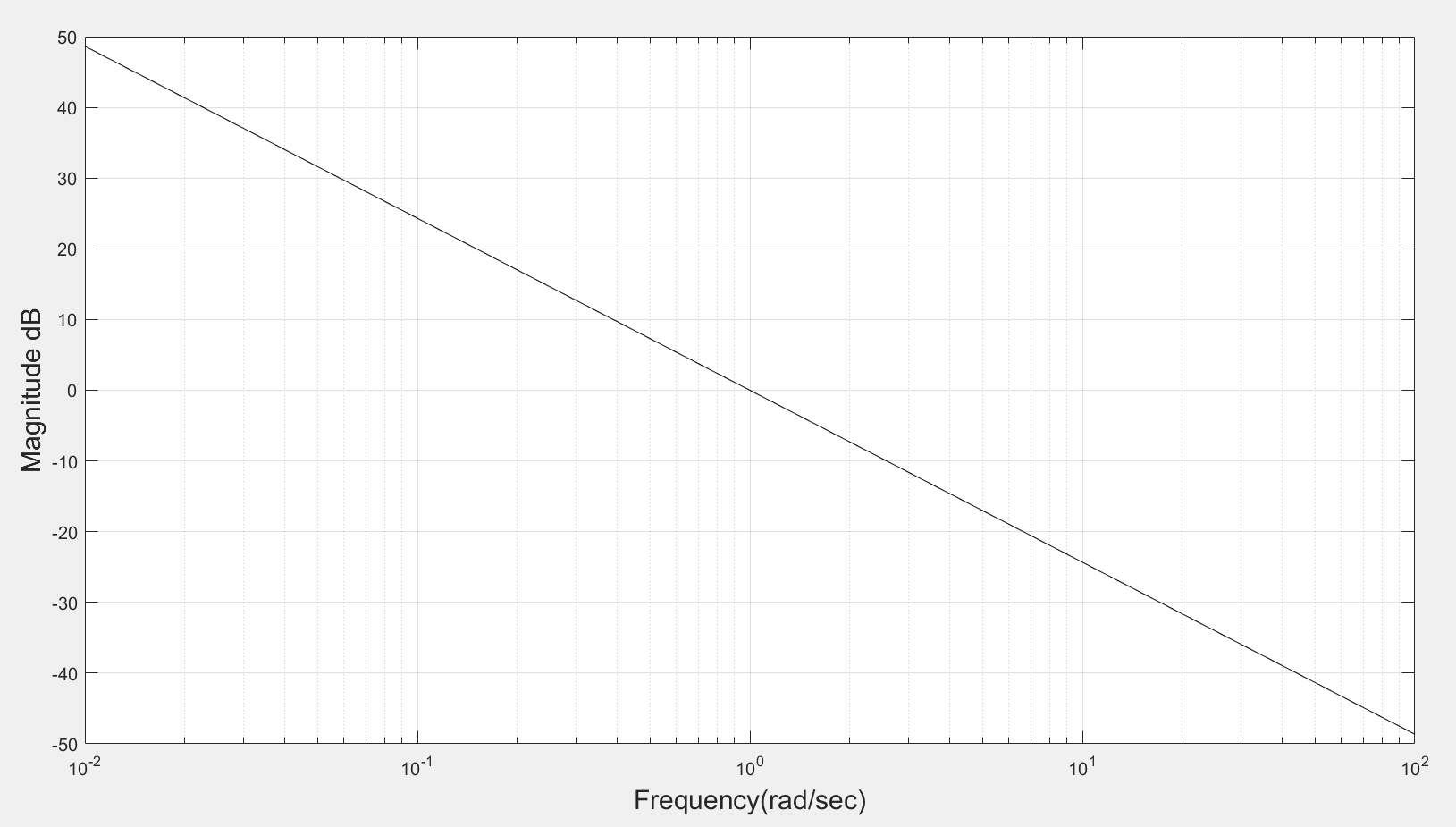


Figure 3: Asymptotic magnitude bode plot for

1. For , putting K=1 and T = 0.4

) = now put s=jw

) =--------(a)

Now,

==

Applying De Moivre’s theorem in above equation we get

= . …… (b)

Putting (b) in (a)

) = =

Magnitude, |) | =

=

=

Now, Magnitude in dB, |) | dB = -20log

In the sum , dominates at lower frequencies whereas dominates at higher frequencies.

For approximation we consider = .We obtain corner frequency, =.

Now, following approximation of magnitude is obtained:

1. For ω ≤ , |T(jω) | dB = -20log = -20log|1|.
2. For ω >, |T(jω) | dB = -20log= -20αlog (0.4.ω)

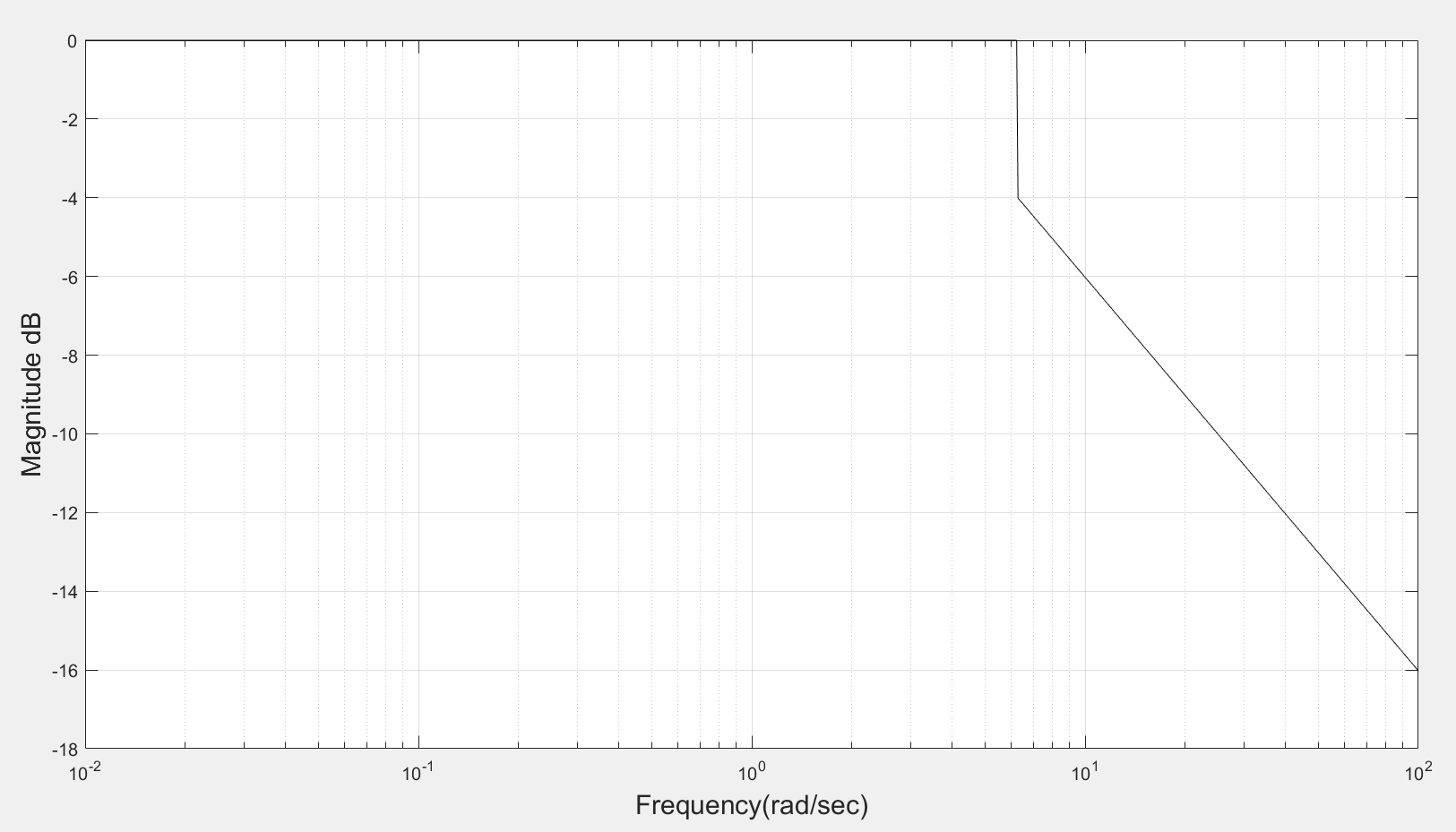


Figure 4: Asymptotic magnitude bode plot for

1. is composed of basic terms, by adding the asymptotic plot of ( ), () and () one can find asymptotic plot for L(s)

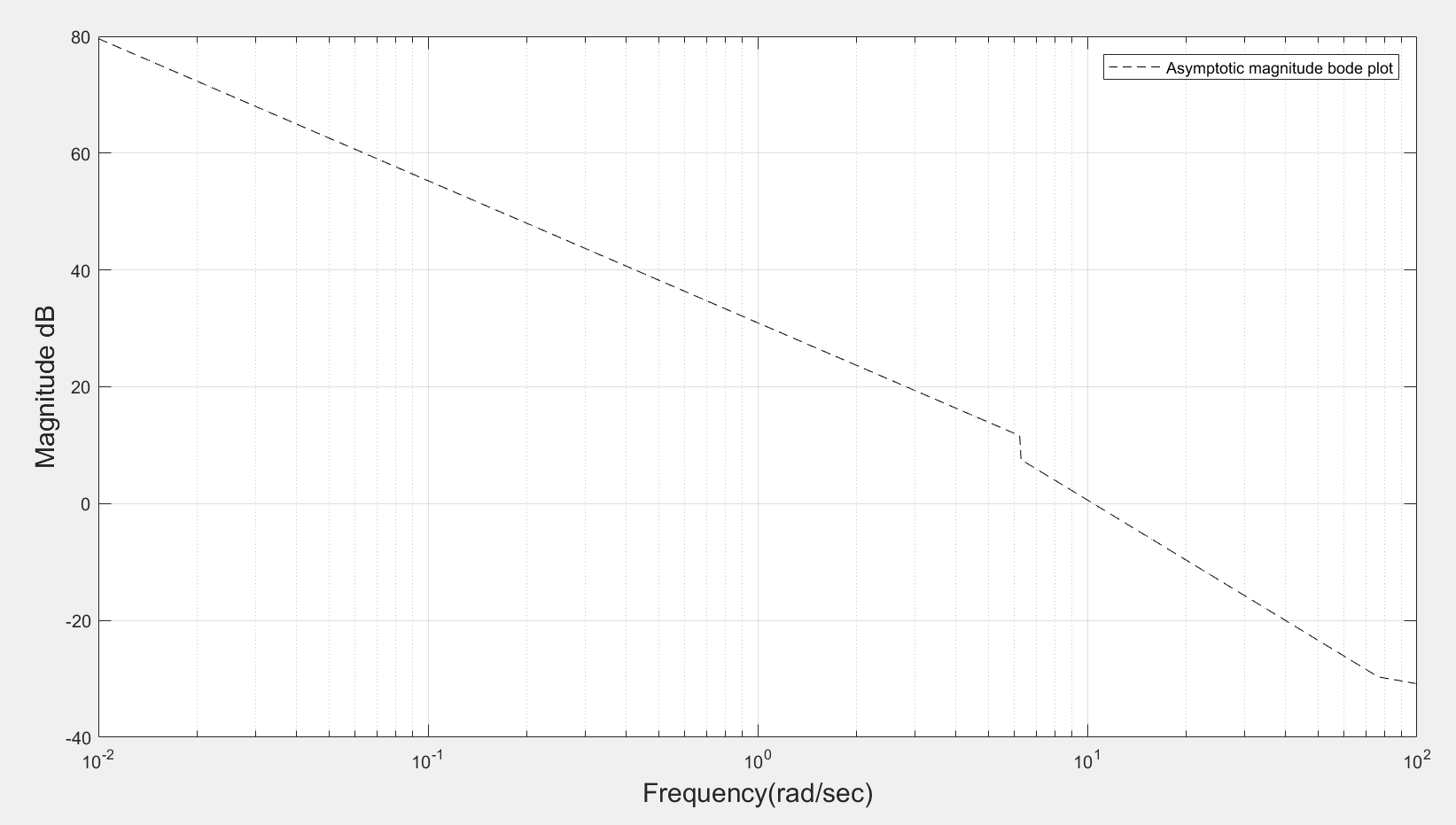


Figure 5: Asymptotic magnitude bode plot for

1. For exact magnitude bode plot of

= C(s)

= () ()

= ()() ( ()

Now,

Magnitude, |)| = 20log

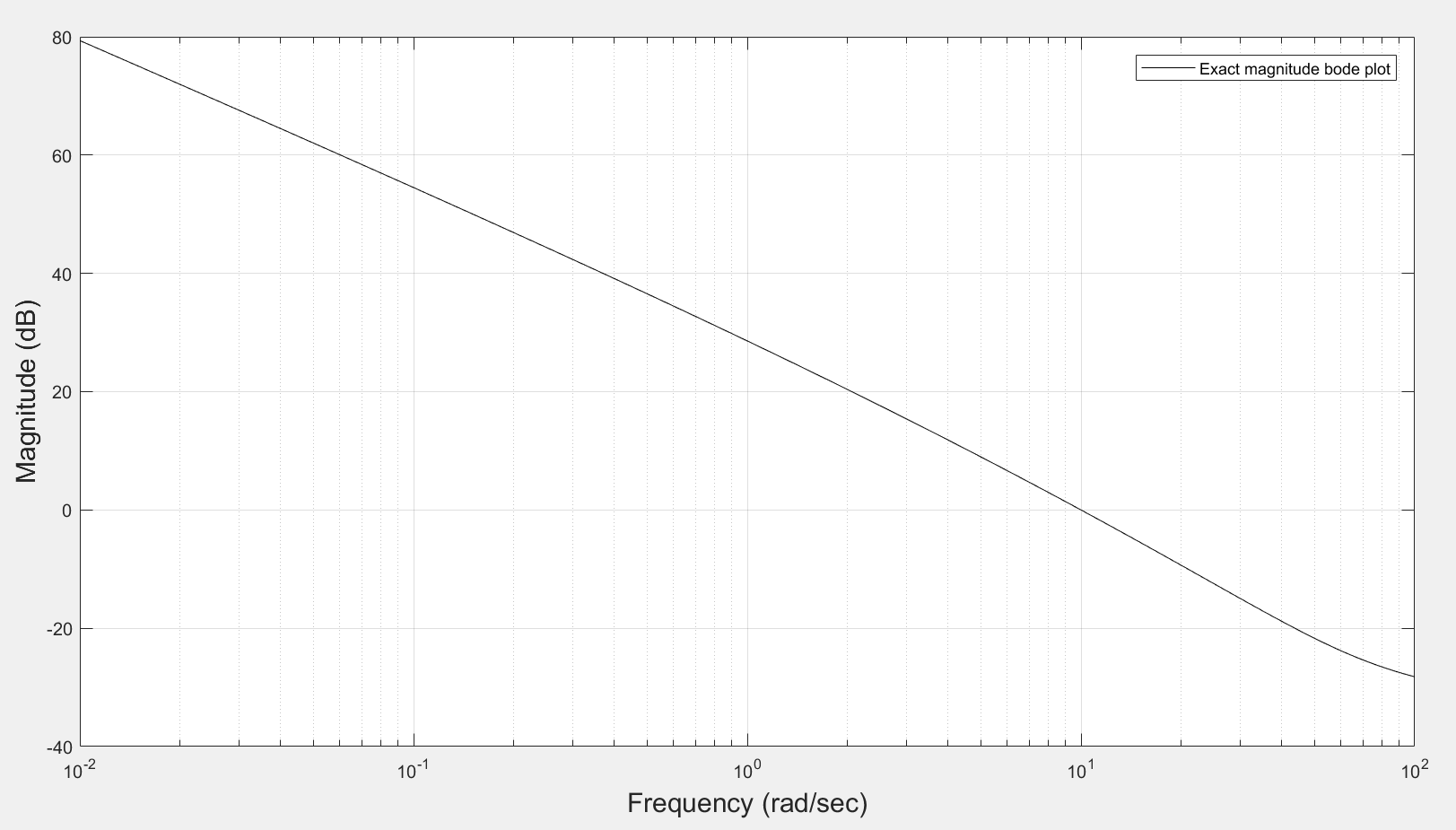


Fig 6: Exact magnitude bode plot for

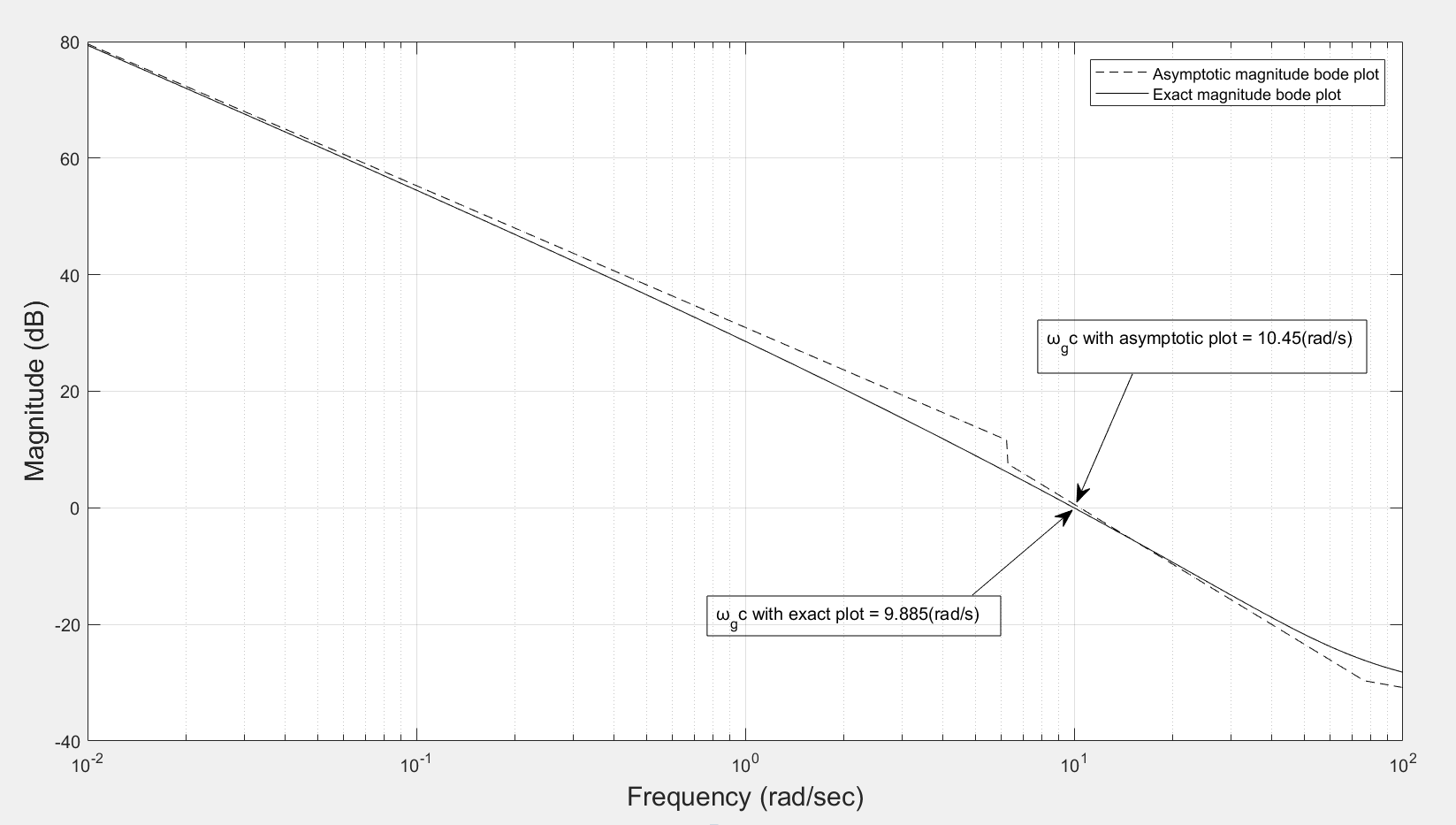


Fig 7: Real and asymptotic magnitude bode plot for

From fig 7, is obtained as:

1. for asymptotic = 10.45(rad/sec)
2. for exact plot = 9.885 (rad/sec)

**Matlab**

clc;

clf;

clear all;

close all;

Kp = 0.1817;

Ki = 194.4;

beta = 1.216;

alpha = 0.5;

Wcr = (abs(Ki))^(1/beta);

Wcr1 = (abs(1/0.4))^(1/alpha);

w = logspace(-2 , 2, 1000);

mask = w < Wcr;

mask1 = w < Wcr1;

mag1 = 20\*log10(Kp)\*ones(size(w));

mag2 = mask.\*(20\*log10(Ki)) + (~mask).\*(20\*beta\*log10(w));

mag3 = -20\*beta\*log10(w);

mag4= (mask1).\*(-20\*log10(1)) + (~mask1).\*(-20\*alpha\*log10(0.4\*w));

combined = mag1 + mag2 + mag3 + mag4;

figure(1);

plot(1,1);

semilogx(w, mag1,'Color','black')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

figure(2);

plot(1,2);

semilogx(w, mag2,'Color','black')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

figure(3);

plot(1,3);

semilogx(w, mag3,'Color','black')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

figure (4);

plot(1,4);

semilogx(w, mag4,'Color','black')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

figure(5);

plot(1,5);

semilogx(w, combined,'Color','black','LineStyle','--')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

legend('Asymptotic magnitude bode plot','Location','NorthEast');

plot(1,1);

semilogx(w, combined,'Color','black','LineStyle','--')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

hold on;

clc;

clear

beta = 1.216;

alpha = 0.5;

K=1;

T=0.4;

Kp=0.1817;

Ki=194.4;

w=logspace(-2,2,1000);

Mag=@(w) 20\*log10(abs(sqrt(K).^2)) + 20\*log10(abs(sqrt(Kp).^2)) + 20\*log10(abs(sqrt((1i.\*w).^beta +Ki).^2)) - 20\*log10(abs(sqrt(T\*(1i.\*w).^alpha + 1).^2)) - 20\*log10(abs(sqrt((1i.\*w).^beta).^2));

semilogx(w,Mag(w),'Color','black');

hold on;

grid on;

xlabel('Frequency (rad/sec)','FontSize', 15);

ylabel('Magnitude (dB)','FontSize', 15);

legend('Exact magnitude bode plot','Location','NorthEast')